SIMULTANEOUS CODED PLANE WAVE IMAGING IN ULTRASOUND: PROBLEM FORMULATION AND CONSTRAINTS

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Context - Motivation

Ultrasound is often used to image fast transient events

Elastography
(@: www.institut-langevin.espci.fr)

Flow imaging
(Courtesy of Alfred Yu,
University Waterloo, Canada)

High frame rate ultrasound could improve such techniques in 2D[1] [2]

Higher acquisition rate of 2D frames => Higher frame rate of 3D frames


Plane wave imaging\cite{1}

- Received signals
- Beamforming
- Log compression
- Final Image

Ultrafast acquisition rate
\( \sim 2 \cdot \frac{\text{depth}}{c} \)

Presence of artefacts

Plane wave coherent compounding imaging [1]

Emission angle ($\theta$)

$\theta = 0^\circ$

$\theta = 4^\circ$

$\theta = -7^\circ$

Our proposal: Simultaneous emission of the plane waves

Q: What is the mathematical model of the system?
Q: What inverse problem approach to use in order to solve the model?
Q: What excitation signals to use?
At the end of this presentation you will know:

The mathematical model of our system

The estimator that we use to solve our system

The conditions under which our system becomes well-posed

The excitation signals that allow the system to be well-conditioned — Codes
Emission/reception of a plane wave carrying an arbitrary signal \( a(t) \)

\( j \)th Emitted signal:

\[
a_j(t) = a(t) \ast \delta(t - t_j)
\]

- \( a(t) \) — signal carried by the plane wave

\[
t_j = (j - 1) \times \text{pitch} \times \tan(\theta)
\]

- \( \theta \) — tilt of the plane’s wave wavefront

\( t_j = 0, \, \forall j \in [1, ..., N_e], \, \theta = 0^\circ \)

\( t_j < t_{j+1}, \, \forall j \in [1, ..., N_e], \, \theta = 4^\circ \)

\( t_j > t_{j+1}, \, \forall j \in [1, ..., N_e], \, \theta = -7^\circ \)
Emission/reception of a plane wave carrying an arbitrary signal \( a(t) \)

**\( i \)th Received signal:**

\[
y_i(t) = \sum_{j=1}^{Ne} a_j(t) \ast h_{je}(t) \ast g_{ji}(t) \ast h_{ir}(t) + v_i(t)
\]

- \( h_{je}(t), h_{ir}(t) \) — acousto-electrical impulse responses of elements at emission, reception
- \( g_{ji}(t) \) — impulse response of the medium when element \( j \) emits and \( i \) receives

\[
y_i(t) = a(t) \ast g_i(t) + v_i(t)
\]

\[
g_i(t) = \sum_{j=1}^{Ne} \delta(t - t_j) \ast h_{je}(t) \ast g_{ji}(t) \ast h_{ir}(t)
\]

\( g_i(t) \) is the pulsed plane wave response of the medium seen by the \( i \)th element of the probe

\( g_i(t) \) — raw signals to be beamformed into the image of the medium
One plane wave model discretization

\[ y_i(t) = a(t) * g_i(t) + v_i(t) \quad \rightarrow \quad y_i = A \times g_i + v_i \]

\[ y_i = [y_i[0] \quad y_i[1] \quad y_i[2] \quad y_i[3] \ldots \quad y_i[N_y]]^T \]

\[ g_i = [g_i[0] \quad g_i[1] \quad g_i[2] \quad g_i[3] \ldots \quad g_i[N_g]]^T \]

\[ v_i = [v_i[0] \quad v_i[1] \quad v_i[2] \quad v_i[3] \ldots \quad v_i[N_y]]^T \]

\[ A = \begin{bmatrix}
    a_{N_a-1} & a_{N_a-2} & \cdots & a_0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
    0 & a_{N_a-1} & \cdots & a_1 & a_0 & \cdots & 0 & 0 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & 0 & \cdots & 0 & 0 & \cdots & a_{N_a-1} & a_{N_a-2} & \cdots & a_0
\end{bmatrix} \]

\( A \) is a \( N_y \) by \( N_y + N_a - 1 \) matrix

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\( g_i \) samples

0 \( \frac{N_a}{2} \) \( \cdots \) \( N_a \) \( \cdots \) \( \frac{3N_a}{2} \) \( \cdots \) \( N_y + N_a \) \( \cdots \) \( N_y + 2N_a - 1 \)
Emission/reception of $N_{pw}$ plane waves carrying signals $a^k(t)$

$$a_j(t) = \sum_{k=1}^{N_{pw}} a^k(t) \ast \delta(t - t^k_j) \quad k \text{ - plane wave index}$$

$$y_i(t) = \sum_{k=1}^{N_{pw}} a^k(t) \ast g^k_i(t) + v_i(t)$$

$$y_i = \sum_{k=1}^{N_{pw}} A^k \times g^k_i + v_i$$

Replacing:

$$A_c = \begin{bmatrix} A^1 & A^2 & A^3 & \cdots & A^{N_{pw}} \end{bmatrix}$$

$$\bar{g}_i = \begin{bmatrix} g^1_i & g^2_i & g^3_i & \cdots & g^{N_{pw}}_i \end{bmatrix}$$

$$y_i = A_c \bar{g}_i + v_i$$

Estimate $\bar{g}_i(t)$ to reconstruct the ultrasound images

For this paper we used Linear Square Estimator:

$$\hat{g}_i = (A_c^T A_c)^{-1} A_c^T y_i$$
Constraints for a well-posed system

\[ A_C = [A^1, A^2, A^3, \ldots, A^{N_{pwii}}] \]

\( A_C \) size: \( N_y \) rows, \( N_{pwii}(N_y + N_a - 1) \) columns

Constraint 1, on the medium:

\[ g_i[z] = 0, \forall z \in [0 \ldots N_a] \cup [N_y + N_a - 1 \ldots N_y + 2N_a - 1] \]

New \( A_C \) size: \( N_y \) rows, \( N_{pwii} \times N_g \) columns
with: \( N_y = N_a + N_g - 1 \)

Constraint 2, on the length of \( a^k(t) \):

\[ N_a = (N_{pwii} - 1)N_g + 1 \]

The emitted signals \( a^k \) must be \( N_{pwii} - 1 \) times longer than the impulse response of the medium
Constraints on the correlation of the emitted signals

\[ \mathbf{A}_c = [\mathbf{A}^1 \quad \mathbf{A}^2], \text{ } N_y \text{ rows by } 2N_g \text{ column matrix} \]

Low mutual coherence of \( \mathbf{A}_c \) => \( \mathbf{A}_c \) well conditionned

Excitation signals: 

\[ a^k(t) = s^k(t) \sin(2\pi f_0 t) \]

\( s^k(t) \) is a pseudo-orthogonal code

Frequency shift to \( f_0 \) using BPSK modulation
Constraints on the correlation of the emitted signals

\[ A_c = \begin{bmatrix} A^1 & A^2 \end{bmatrix}, \text{ } N_y \text{ rows by } 2N_g \text{ column matrix} \]

\[ a^1(t) \text{ auto-corr.} \quad a^2(t) \text{ cross-corr.} \]
FieldII$[1][2]$ simulations results

\[ \tau = \frac{N_{@5pwi} - N_y}{N_{@5pwi}} \times 100\% \] — time gain obtained with coded approach

Reducing the observation time adds a Perturbation Zone, that makes the system ill-posed thus the CNR drops

Conclusion

A mathematical model of the simultaneous coded plane wave imaging

The physical constraints on the medium and emitted signals for a well posed system

Validation of the imaging process model without time gain

Increasing the frame rate leads to image quality decrease

Q: What’s next?

Implement regularization to solve the inverse problem
Thank you!